

Exercise 1

Reduce each of these quantities to a real number:

$$(a) \frac{1+2i}{3-4i} + \frac{2-i}{5i}; \quad (b) \frac{5i}{(1-i)(2-i)(3-i)}; \quad (c) (1-i)^4.$$

$$\text{Ans. (a) } -2/5 \quad (b) \quad -1/2; \quad (c) \quad -4.$$

Solution**Part (a)**

$$\begin{aligned} \frac{1+2i}{3-4i} + \frac{2-i}{5i} &= \frac{(1+2i)(3+4i)}{(3-4i)(3+4i)} + \frac{(2-i)(-5i)}{(5i)(-5i)} \\ &= \frac{(1+2i)(3+4i)}{9-16i^2} + \frac{-10i+5i^2}{-25i^2} \\ &= \frac{3+4i+6i+8i^2}{25} + \frac{-10i-5}{25} \\ &= \frac{3+10i-8-10i-5}{25} \\ &= \frac{-10}{25} \\ &= -\frac{2}{5} \end{aligned}$$

Part (b)

$$\begin{aligned} \frac{5i}{(1-i)(2-i)(3-i)} &= \frac{5i}{(2-i-2i+i^2)(3-i)} \\ &= \frac{5i}{(2-3i-1)(3-i)} \\ &= \frac{5i}{(1-3i)(3-i)} \\ &= \frac{5i}{3-i-9i+3i^2} \\ &= \frac{5i}{3-10i-3} \\ &= \frac{5i}{-10i} \\ &= -\frac{1}{2} \end{aligned}$$

Part (c)

$$\begin{aligned}(1 - i)^4 &= (1 - i)(1 - i)(1 - i)(1 - i) \\ &= (1 - 2i + i^2)(1 - 2i + i^2) \\ &= (-2i)(-2i) \\ &= 4i^2 \\ &= -4\end{aligned}$$